Encouraging Perseverance in Elementary Mathematics:  
The Tale of Two Problems

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Abstract: We have probably all seen children who are reluctant to continue to work with a mathematics problem that is not solved easily. In this article, we describe the work of a second-grade class on two challenging, but engaging problems. Although the problems both took several days to solve, the atmosphere of persistence and perseverance that the teacher had created in the classroom was such that the students relished the challenge and were delighted with what they found.

Students’ beliefs about mathematics—Shaping behavior in the classroom

Professor Alan Schoenfeld summarized his own and others’ findings on the beliefs about mathematics that are held by many students. They included the following:

- Mathematics problems have one and only one right answer.
- There is only one correct way to solve any mathematics problem--usually the rule the teacher has most recently demonstrated to the class.
- Ordinary students cannot be expected to understand mathematics; they expect simply to memorise it and apply what they have learned mechanically.
- Mathematics is a solitary activity, done by individuals in isolation.
- Students who have understood the mathematics they have studied will be able to solve any problem in five minutes or less. (p. 359)

Schoenfeld claimed that students abstract their beliefs about formal mathematics in large measure from their experiences in the classroom, and that these beliefs “shape their behavior in ways that have extraordinarily powerful (and often negative) consequences” (p. 359).

He cites his own research (Schoenfeld, 1988) involving a survey of 227 high school mathematics students in Grades 9 to 12, who when asked “if you understand the materials, how long should it take

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to answer a typical homework problem?” gave answers that averaged 2.2 minutes. The same students, in response to the question, “what is a reasonable amount of time to work on a problem before you know it’s impossible?” gave responses which averaged 11.7 minutes.

Although these were high school students, if these beliefs are largely shaped by classroom experiences, then patterns presumably begin to be established in the early years of elementary school. Given that most readers would be concerned about these students’ views about the discipline of mathematics, and what it means to do mathematics, then it could be argued that teachers in all grade levels have a responsibility, by their words and actions to present a different view.

So what can the typical classroom teacher do? Several authors (e.g., Bird, 1999; Folkson, 1995) have illustrated how the use of challenging and engaging problems can impact on children’s beliefs about mathematics and themselves as learners.

One strategy is for the teacher to model the struggle often involved in solving genuine problems. The teacher can introduce a problem that she/he has not yet solved, and show students the kind of process involved in making progress on such a problem. Just the act of showing students that the teacher cannot solve a particular problem immediately will be most informative for many children who may have thought that teachers can solve and have solved all problems that have ever been created!

In the remainder of this article, we want to share the experience of a class of second-graders, as they “wrestled” with two problems in the supportive yet challenging environment created by a wonderful teacher, and saw the benefits of the struggle, achieving worthwhile solutions and considerable satisfaction from their efforts.

Background
The classroom experiences discussed in this paper took place as part of the Early Numeracy Research Project (ENRP) in Victoria, Australia. 350 K-2 teachers in 35 schools participated in a three-year research and professional development project, exploring the most effective approaches to the teaching of mathematics in the first three years of school. There were three key components of this project:

- a research-based framework of "growth points" in young children's mathematical learning (in Number, Measurement and Geometry), highlighting typical learning trajectories and key stepping stones in children’s thinking and strategies;
- a 40-minute, one-on-one interview, used by all teachers with all children at the beginning and end of the school year (at the time of writing the interview had been used with over 36,000 children at K-4);
extensive professional development at central, regional and school levels, for all teachers, coordinators, and principals, with the focus on taking what was learned from the interview and day-to-day interactions with children to inform planning and teaching for maximum effectiveness, both cognitive and affective.

Further information on the project can be found in Clarke (2001), Clarke et al., (2002), and Clarke, Cheeseman, McDonough & Clarke (2003).

As part of the professional development for the project, the research team made 578 visits to schools, working with teachers, children, principals, mathematics coordinators, and parents. A common feature of these visits involved working in classrooms, either joining in with the regular mathematics activities of the class, or team teaching with the regular classroom teacher, often trying particular mathematics tasks and problems that had not been used before by the researchers or teachers. The following discussion concerns two such problems.

The First Problem

Ararat North Primary School is a small country school in Victoria, in a farming community. During a school visit, one of the authors presented Anne Joyce’s Grade 2 class with an activity that led to a problem being posed. The activity was adapted from the *Primary Initiatives in Mathematics Education* materials from England (see Shuard, 1992).

The experiences described here took place in the latter part of the school year, when most Grade 2 children are around eight years old.

Following a whole group introduction, children were given cardboard triangles, a pile of unifix cubes, and small cards with the numbers from 1 to 20.

Working in groups of three, children were asked to shuffle the cards. Then, in turn, they were to turn over a card, take that number of unifix cubes, and build three towers, one in each corner of the triangle.

Children were asked to make the three towers as close in height to each other as possible. For each number turned over, they were asked to
record in some way whether the towers could all be the same height or not. So, for example, 12 cubes would work, but 7 would not. As we see in the picture, sharing 4 would lead to unequal towers once the last cube is placed.

As the children worked, the teacher and the researcher moved around the room, encouraging children to share what they had noticed to that point.

One interesting note was that some children had hypothesized that even numbers of cubes would lead to “even towers,” a nice example of how the meaning of a word in everyday language (meaning about the same), can be somewhat different from its mathematical meaning.

After about thirty minutes, we brought the class back together again, and groups shared what they had found. The table on the whiteboard summarized some of what they had found so far.

Children were invited to make conjectures about any patterns that were evident, but apart from one child
pointing out that there were counter-examples to the theory that even numbers gave even towers, no others patterns were suggested. In discussions with the teachers later, we noted that this was further evidence to that which emerged in the one-to-one interviews of the challenges that concepts related to multiplicative thinking present to young children. It is worth noting that we made the deliberate decision not to put the left-hand column into numerical order at this stage. The temptation for the teacher at this stage is to attempt to bring closure to the activity by leading the children (by the nose) to the desired pattern. However, Anne Joyce resisted this temptation, and encouraged the children to continue to think about possible patterns over the coming days. She said, “each time you walk past the whiteboard over the next couple of days, see if you can see anything interesting there, and let me know what you find.”

A week later, all of the project teachers met in the city, for a full-day’s professional development. Anne Joyce had brought something for the authors from the children. She explained that the children had continued to consider the patterns in the findings (if any), and had kept a record, with different children writing parts of the story. She left this story with us.

At this point, we will let the children share the process they went through.

When we worked with Mr Clarke he asked us to play a game with numbers and a triangle.

We made up a yes/no chart about equal towers on each corner.

We decided that zero was a ‘yes.’

One person thought evens might make equal towers and odds wouldn’t. When we looked at the yes/no columns we found there were odd and even numbers in each one.

We re-arranged the ‘yes’ numbers and found they were all the third numbers if we started at zero.
The authors were most excited with what the children had discovered, but also the way in which they had taken the original problem, and attempted to try it for different shapes, leading to a level of generalization. Their persistence in continuing to work on what was for them a challenging problem was also a highlight.
The positive experience of working on this problem encouraged the authors to send the children another problem.

**The Second Problem**

A number of years ago, we came across the following problem (source unknown):

*A man goes into a store and says to the owner: “give me as much money as I have with me and I will spend $10”. It is done, and the man does the same thing in a second and third store, after which he has no money left. How much did he start with?*

We invite the reader to take some time to work through this problem. We confess readily that this is a bit of a silly problem, but we have found it very useful over the years in our work with preservice teachers, in classroom sessions relating to problem solving strategies. Of course, it is one thing to present it to preservice teachers and quite another altogether to present it to eight-year olds. Nevertheless, we faxed a copy of the problem to the children, explaining that it was extremely difficult, but that we thought they might relish the challenge it provided.

We heard nothing from the school for about three weeks, until a large poster arrived late one Friday afternoon. The accompanying note from Anne Joyce explained that they had continued to revisit the problem over a number of days, once again sharing the task of recording the thinking and discoveries at each stage on the blackboard. These were then written down and sent to us.

Once again, we will let the children tell the story of their work, with words and pictures, and our occasional comments:

We read the problem and thought hard.

Most people thought $30, so we made lots of notes and acted out the problem with one man and three shop owners.

We were pleased to see that the children were encouraged to come up with an initial estimate, and it was interesting that most
people thought $30, presumably because three times $10 is $30.

It was also interesting that they had decided that acting out the story was a way of making sense of what the problem was actually asking. Physical involvement is often a powerful tool in the mathematics classroom.

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\begin{align*}
20 & + 20 = 40 - 10 = 30 \\
30 & + 30 = 60 - 10 = 50 \\
50 & + 50 = 100 - 10 = 90
\end{align*}
\]

Of course, it would be important to point out to the children that the use of the equals sign in their various statements is incorrect, emphasising to them that both sides of these “equations” are not equivalent.

The children continued to try different values, but each time they were ending up with a relatively large final figure.

\[
\begin{align*}
15 & + 15 = 30 - 10 = 20 \\
20 & + 20 = 40 - 10 = 30 \\
30 & + 30 = 60 - 10 = 50
\end{align*}
\]

We saw that as we went along with each problem the numbers were getting bigger.

All the numbers are too big — well try 10

\[
\begin{align*}
10 & + 10 = 20 - 10 = 10 \\
10 & + 10 = 20 - 10 = 10 \\
10 & + 10 = 20 - 10 = 10
\end{align*}
\]

We are standing still.

We liked the comment that “we are standing still.” At this stage, the children realized that they still needed to start with a smaller starting number:

Ten is the closest so far but we need less left over.
These children haven’t had our experience of the world to realize that the final sentence is not necessarily true!

But then came a crucial point in the children’s problem solving process, both in the discovery of what happens if you start with $5, and on the strategy of working backwards.

The children were now “on their way,” and although the arithmetic was complicated, particularly for Grade 2 children, after much effort, a solution was reached.
Some readers may be doubtful that typical second graders could do such work, and so it is worth noting an accompanying comment from Anne to the authors. She explained that after a while, the mathematics got too difficult for some children. Nevertheless, they all continued to participate in various ways, making the notes, acting out the role of storekeeper, and so on, while those children who could cope with the increasingly complex mathematics, continued to pursue a solution.

The children concluded their summary with the following statement:

\[
\text{It took a lot of thinking and working out.}
\]

Of course, we were thrilled with the quality of the students’ thinking and persistence, with such a difficult problem.

**Grade 2 Children as Mathematical Thinkers**

In an excellent discussion of mathematical thinking, Watson and Mason (1998), list 19 “kinds of mental activity which, together, typify mathematical thinking” (p. 7). It could be argued that, of these, the following fourteen were evident in the children’s work on these two problems:
There is no question that these children were engaged in mathematical thinking of an impressive kind, providing great personal satisfaction for themselves and their teacher. In this way, as well as children being exposed to what it means to work mathematically, there will be a positive influence on affective aspects as well.

In Conclusion

We began this article by talking about the worrying beliefs about the nature of mathematics and doing mathematics that develop in many students during their time in mathematics classes at school. In particular, there is the concern that students believe that if problems can’t be solved almost immediately, they are impossible.

As this story illustrates, if teachers choose rich problems for use with children, and they encourage persistence, working together, making conjectures, sharing their findings, and allowing time for possibilities to emerge, then there is a considerable chance that such beliefs will in time be less evident in the later years. Hopefully, such beliefs will be replaced by a confidence that as they are presented with difficult mathematics problems, requiring ongoing struggle and persistence, that considerable satisfaction and pleasure can be derived from solving challenging problems of this kind.

References


Clarke, Doug M. “Understanding, Assessing and Developing Young Children’s


